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Inventory Models with Lost Sales and Random Deterioration

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Abstract— During the course of this research, the instantaneous economic order quantity model is investigated. This model entails calculating the proportion of units that are lost as a result of deterioration in an existing inventory. In addition to this, it takes into account the expenses that are connected to promotional efforts and changeable ordering. Through the process of determining, the goal is to maximize the net profit that is generated from the transaction, taking into account the quantity ordered, the factor of promotional activity, the length of the cycle, and the quantity of units lost to deterioration. It has been demonstrated through research that there is a single perfect replenishment plan that can be universally applied to whatever number of replenishment cycles that are being studied. The net profit function's concavity is also taken into consideration when developing a mathematical model for the purpose of determining important factors associated to it. A number of numerical examples that illustrate the advantages for the retailer are included in the model that has been proposed. It is very important to implement this strategy in order to solve the problem of waste for things that are deteriorating. In addition, the ideal solution is put through sensitivity studies in relation to the main parameters, and a comparative analysis of a large number of EOQ models that are related to it is carried out.

Index Terms: Inventory Models, Lost Sales, Random Deterioration, Stochastic Inventory Control, Dynamic Programming, Cost Efficiency, Service Levels.

I. INTRODUCTION

As far as the business sector is concerned, inventory is a requirement that cannot be avoided. In a great number of contemporary businesses, it is an indispensable component. In spite of the fact that it is a resource-intensive and expensive procedure, maintaining an inventory helps to ensure that a consistent flow of products is delivered to clients. This is the ideal circumstance for any company, as it eliminates the need for inventory. This guarantees that the vendor will deliver the product at the precise moment when the purchaser demands it.

It is considered a current asset on a company's balance sheet because it can be swiftly converted into cash through sales. This is the reason why inventory is reported as a current asset. Some businesses keep larger inventories than what is actually required for their operations in order to artificially inflate their perceived asset value and profitability. This is done in order to achieve this manipulation. In certain instances, the goods that the firm considers to be inventory are also the things that it uses to run its business.

In order for organizations to maximize the use of the resources they have at their disposal, effective inventory management is an essential component of an organization's operations. The following types of resources are included in this category: manpower, tools, capital, material property, and data. A corporation's ability to control these elements is directly proportional to the degree to which it is competent with other companies. The mathematical models that have been developed have provided solutions for the management of these resources.

It is a fundamental obligation for any organization in India, Whether it's a government agency, a business in the public or private sector, the goal is to maximize the nation's few resources to provide products and services. In order to meet this expense, the majority of the money that is required comes from the materials, and the majority of the working capital comes from the inventory. Consequently, being able to regulate inventories and effectively manage supplies are essential components of effective productivity management. It is possible for supervisors to assist in ensuring that supplies are utilized effectively and that working capital is handled effectively by using rigorous inventory management.

Inventory is comprised of a range of commodities, including as components, work-in-process (WIP), completed items, raw materials, package materials, and general supplies. Any owner or stage of the product can reap the benefits of these stock reasons.

A. Importance of inventory management

(i) To prevent delivery delays;
(ii) To keep production flowing smoothly and efficiently;
(iii) To keep better relations with customers;
(iv) To take advantage of quantity discounts;
(v) To take advantage of price fluctuations;
(vi) To prevent materials from being scarce in the market; and
(vii) To make better use of human machinery.

In today's fast-paced and competitive corporate environment, effective inventory management has become an essential performance indicator that must be attained. Businesses need to figure out how to strike a balance between the costs of their inventory, the demand from customers, and the quality of their products if they want to achieve maximum efficiency. Among the many challenges that inventory managers face, two important problems that can have a significant impact on profitability and customer satisfaction are sales that are lost due to stockouts and the random deterioration of inventory goods. Both of these issues can have a significant impact on the overall satisfaction of customers.

When there is not enough inventory on hand to satisfy the requirements of the market, sales opportunities are lost. It is possible that this will strain connections with customers and undermine their dedication to the brand, in addition to the evident decline in sales that would occur as a result of this occurrence. Management strategies that are effective must take into account both the chance of missed sales and the impact of those missed sales. This necessitates the use of complex models that can both predict and reduce the likelihood of such hazards occurring.

In addition to effectively managing supply and demand, businesses must also contend with the unpredictable deterioration of commodities over time. One phenomenon that is commonly called "random deterioration," unexpected wear and tear on inventory products can lead to an increase in expenses, a decrease in quality, and a reduction in profits. This component makes inventory management more challenging because of the inherent unpredictability that is associated with product shelf life.

Loss of sales and random deterioration both contribute to an increase in the complexity of inventory management. Considering that conventional models typically focus on just one of these two concerns by themselves, To address the combined impact of these two issues on inventory systems, a more all-encompassing approach is required. In order to shed light on how well these models operate and how useful they are in practical settings, this study aims to contrast and compare different inventory models that account for missed sales and random deterioration. This research will be conducted in order to do this.

II. LITERATURE REVIEW

M. Valliathal (2023), This study analyses the two-warehouse inventory model for deteriorating items with shortages using an infinite-time horizon economic order quantity technique. Shortages are allowed, and some demand is backlogged. The backlog rate is part of the inventory. We analyze demand over time. This strategy reduces retailer costs. For model and solution verification, numerical examples are supplied. Complete and time-dependent partial backlogging models are compared.

T. Vijayan (2018), Periodic and ongoing reviews under fuzzy environments, inventory models are taken into consideration where a portion of demand is backordered and the remaining portion is lost during the stockouts period. Allowing the cost components to be somewhat ambiguous and imprecise introduces fuzziness. These features are represented as trapezoidal fuzzy numbers. We derive the optimal strategies of these models under fuzzy costs. There is also a description of numerical results that show how sensitive the choice variables are.

Biswaranjan Mandal (2021), In our daily lives, demand for fruits, flowers, green vegetables, dairy products, and the like is high, but it is declining because these goods are stored in cold storage facilities, farms, flower shops, and supermarkets, where they may rot or decay. We can't ignore how improvement and degradation effect inventory management. The premise of a constant demand rate may not apply to inventory commodities like milk and vegetables. Because consumers lose faith in product quality, older inventory lowers demand. This model allows entirely backlogged shortages and views demand as a cubic function of time. Solving the model uses salvage values from decaying units across the cycle. Vendor-customer cooperation is the most important part of inventory management. In many inventory instances, customers have a grace period to repay their purchases without interest. Acceptable payment delays are a win-win strategy for cooperative profit sharing. The buyer may receive interest on inventory sales during the payment period. An inventory model with time-varying deteriorating products and two-parameter ameliorating commodities with cubic demand under salvage and permitted payment delays is discussed in this study. Last, numerical examples show the concept and certain specific cases.

Paul H. Zipkin (2018), We considered the notoriously difficult discrete-time inventory model with lost sales, constant lead time, and stochastic inquiries. We show that the effective state space is confined and controlled. Next, we analyze some reasonable guesses. Despite their flaws, some perform well. The best policy for a backlog system is almost identical to the base-stock method, which fails. Additionally, we show that lead time raises the optimal cost.

N L Devy (2018), This study addresses the combined replenishment inventory problem of two goods with predictable and steady demand. Inventory items are refilled at T time intervals. Both products can be replenished together. Item I is refilled per ZiT time interval. Products replenish instantly. All shortages count as lost sales. Lost sales for item I are limited to Si. A mathematical model determines the basic time cycle T, replenishment multiplier Zi, and maximum lost sales Si to reduce cost per unit time. A solution method is offered and a numerical example is given to demonstrate its usefulness.

Gede Agus Widyadana (2020), Many researchers and practitioners are developing and using the economic production quantity (EPQ) model. This study seeks to construct more realistic EPQ models for degrading items by considering price-dependent demand and uniformly and exponentially distributed stochastic machine unavailability time. Sales are lost when a machine's downtime exceeds its production time. Since closed-form solutions are impossible, we use the Genetic Algorithm (GA) to solve the models. The models are shown with sensitivity analysis and numerical examples. The sensitivity tests show that management can utilize pricing strategy to limit profit loss from machine unavailability time when demand is based on price.

A. Notation and Assumptions

r: Calculate of consumption

t_c: Repeat duration

h: retaining a single unit's cost for a single time unit.

HC (q): keeping cycle costs in place

C: Unit purchasing expenses

Ps: The unit selling price

α: the proportion of available inventory lost due to degradation

Q: The whole order amount

$K * q^{r-1}$: Cost of ordering each cycle (OC): $0 < \gamma < 1$

q**: Economic ordering and modified production quantity (EOQ/EPQ)

q*: in conventional terminology, the economic ordering quantity, or EOQ

$\varphi(t)$: Level of inventory on hand at moment t

ρ: The factor of promotional effort for each cycle.

PE^{ρ}: The expense of the promotional campaign, $PE^{\rho} = K_1(\rho-1)^2 r^{\alpha_1}$ where α_1 is a constant and $K_1>0$.

π1 (q, ρ): A simple approach is used to generate q units in a single cycle in order to compute the overall profit per unit.

The average profit per unit for the crisp technique while generating q units in a cycle is π (q, ρ).

B. Mathematical Model

To analyze the inventory level at a specific moment (t) denoted as $\varphi(t)$, let's examine the situation. For a brief period of time between (t) and (t + dt), where t + dt is a later time than (t), From υ(t) to υ(t + dt), the inventory reduces. This reduction can be observed as time passes. To describe the function ϕ(t + dt), we can express the equation in the following way:

$$\varphi(t+dt) = \varphi(t) - r\rho dt - \alpha\varphi(t) \quad (1)$$

The first equation can be rewritten as being:

$$\frac{\varphi(t+dt)-\varphi(t)}{dt} = -r\rho - \alpha\varphi(t) \quad (2)$$

When dt approaches zero, equation (2) becomes simpler to:

$$\frac{d\varphi(t)}{dt} + \alpha\varphi(t) + r\rho = 0 \quad (3)$$

Equation (3) contains a differential equation, the answer to which is

$$\varphi(t) = -\frac{r\rho}{\alpha} + \left(q + \frac{r\rho}{\alpha}\right)e^{-\alpha t} \quad (4)$$

We need to consider the quantity that is promptly restocked in the beginning of every cycle, which lasts for Time units expressed in (t_c). Let (q) represent this replenished quantity. If all units are exhausted due to external demand and deterioration, an immediate restocking of (q) units occurs. By substituting (t_c) into equation (4), we can determine the cycle length, denoted by (t_c). This is achieved by setting the value to zero in the equation.

$$t_c = \frac{1}{\alpha}\ln\left[\frac{\alpha q + r\rho}{r\rho}\right] \quad (5)$$

Equations (4) and (5) are utilized in the process of developing the mathematical model. It is essential to take note of the fact that when the value of α is getting closer t_c gets closer to zero. The overall number of units lost during the course of each cycle, represented by the variable L, can be mathematically defined as follows:

$$L = r\rho\left[\frac{q}{r\rho} - \frac{1}{\alpha}\ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right] \quad (6)$$

Costs associated with ordering and buying each cycle are the two main components that we add together to get the total cost per cycle, or TC (q). We also take into consideration the cost of holding every cycle, which is represented by HC(q) and the cycle-by-cycle expense of promotional effort, which is represented by PE(ρ). From equation (4), the expression for HC(q) is obtained.

$$HC_q = \int_0^{t_c} h\varphi(t)\,dt$$

$$HC_q = h\int_0^{\frac{1}{\alpha}\ln\left[\frac{\alpha q + r\rho}{r\rho}\right]} \left(-\frac{r\rho}{\alpha} + \left(q + \frac{r\rho}{\alpha}\right)e^{-\alpha t}\right)dt$$

$$HC_q = h\left[\frac{q}{\alpha} - \frac{r\rho}{\alpha^2}\ln\left(\frac{rq + r\rho}{r\rho}\right)\right] \quad (7)$$

$$PE(\rho) = K_1(\rho-1)^2 r^{\alpha_1} \quad (8)$$

$$TC = OC + PC + HC + PE$$

$$TC(q,\rho) = Kq^{(\gamma-1)} + cq + h\left[\frac{q}{\alpha} - \frac{r\rho}{\alpha^2}\ln\left(\frac{rq + r\rho}{r\rho}\right)\right] + K_1(\rho-1)^2 r^{\alpha_1} \quad (9)$$

The expression for the total cost per unit of time, or TCU (q,ρ), may be created by dividing equation (9) by equation (5).

$$TCU(q,\rho) = \left\{Kq^{(\gamma-1)} + cq + h\left[\frac{q}{\alpha} - \frac{r\rho}{\alpha^2}\ln\left(\frac{rq + r\rho}{r\rho}\right)\right] + K_1(\rho-1)^2 r^{\alpha_1}\right]\} * \left\{\frac{1}{\alpha}\ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right\}^{-1}$$

$$TCU(q,\rho) = \frac{Kq^{(\gamma-1)}\alpha + (c\alpha + h)q}{\ln\left(1 + \frac{\alpha q}{r\rho}\right)} - \frac{hr\rho}{\alpha} + \frac{K_1(\rho-1)^2 r^{\alpha_1}}{\ln\left(1 + \frac{\alpha q}{r\rho}\right)} \quad (10)$$

As the value of α approaches zero and ρ is equal to 1, equation (10) simplifies to

$$TCU(q) = \frac{Kq^{(\gamma-1)}r}{q} + cr + \frac{hq}{2}$$

Whose answer the conventional EOQ calculation yields, $q^* = \left[\frac{h}{2Kr(2-\gamma)}\right]^{\frac{1}{\gamma-3}}$. The overall gain made from every cycle is

$$\pi_1(q,\rho)\pi_1(q,\rho) = (q - L)$$

$$= (q-L)P_s - Kq^{\gamma-1} - cq - h\left[\frac{q}{\alpha} - \frac{r\rho}{\alpha^2}\ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right]$$

$$-K_1(\rho-1)^2 r^{\alpha_1} \quad (11)$$

Equations (6) and (9) based on the previously given formulae are used to calculate the values for LLL, which stands for the total cost each cycle denoted by TC(q,ρ), and the number of units lost per cycle due to deterioration. Finding the average profit π(q,ρ) per unit of time involves dividing the total cost per cycle (t_c) by the profit function π1(q,ρ). You will receive the average profit for a certain time period using this method. To the greatest extent feasible, the goal is to maximize profit.

$$\text{maximize}\{\pi_1(q,\rho)\}\ \forall\, q > 0, \rho > 0 \tag{12}$$

C. Procedure

For each cycle, the optimal ordering quantity (q) and promotional effort (ρ) must be determined by computing and setting the derivatives of equation (12) with respect to q and ρ to zero. To ensure the uniqueness of the solution, it is necessary to demonstrate that the net profit function is jointly concave with respect to the ordering quantity q and the promotional effort ρ throughout the whole cycle. Moreover, the second partial derivatives of equation (12) with respect to ρ and q must likewise be negative if the determinant of the Hessian matrix is positive. We now look at the following claims.

Proposition 1: There is concavity with respect to q in the net profit π1(q, ρ) each cycle.

Criteria for achieving the most favorable or advantageous state of q

$$\frac{\partial\pi_1(q,\rho)}{\partial q} = \frac{r\rho}{(\alpha q + r\rho)\alpha}(\alpha P_s + h) - \left[K(\gamma-1)q^{(\gamma-2)} + c + \frac{h}{\alpha}\right] = 0 \tag{13}$$

The following expression illustrates the net profit per cycles partial derivative second order in relation to q is as follows:

$$\frac{\partial^2\pi_1(q,\rho)}{\partial q^2} = -\frac{r\rho}{(\alpha q + r\rho)\alpha}(\alpha P_s + h) - [\,K(\gamma-1)(\gamma-2)q^{\gamma-3} \tag{14}$$

Given that $r\rho > 0$

$$(\gamma-1)(\gamma-2) > 0 \ and\ (\alpha P_s + h\alpha) > 0$$

Proposition 2: Every cycle's net profit π1(q, ρ) shows concavity with respect to ρ.

$$\frac{\partial\pi_1(q,\rho)}{\partial \rho} = \left\{\frac{1}{\alpha}\ln\left(\frac{\alpha q}{r\rho} + 1\right) - \left(\frac{\alpha}{\alpha q + r\rho}\right)\right\}\left\{\frac{r}{\alpha}(\alpha P_s + h) - 2K_1(\rho - 1)r^{\alpha_1} = 0 \tag{15}\right.$$

Here, "second order partial derivative" refers to the net profit per cycle's partial derivative in relation to ρ.

$$\frac{\partial^2\pi_1(q,\rho)}{\partial q^2} = -\frac{rq^2}{(r\rho + \alpha q)^2}(\alpha P_s + h) - 2K_1 r^{\alpha_1} \tag{16}$$

Considering

$$(\alpha P_s + h\alpha) > 0, K_1 > 0, r > 0$$

The equation (16) has been found to have a value that is negative, as was determined.

Equation (12)'s second partial derivatives in relation to q and ρ each yield a strictly negative value, as demonstrated by propositions 1 and 2. It is therefore crucial to confirm that the Hessian matrix's determinant is positive.

$$\frac{\partial^2\pi_1(q,\rho)}{\partial q^2} * \frac{\partial^2\pi_1(q,\rho)}{\partial q^2} - \left(\frac{\partial^2\pi_1(q,\rho)}{\partial q\,\partial \rho}\right)^2 > 0 \tag{17}$$

$\frac{\partial^2\pi_1(q,\rho)}{\partial q^2}$ & $\frac{\partial^2\pi_1(q,\rho)}{\partial q^2}$ As demonstrated in Equations (13 and 15) and

$$\frac{\partial^2\pi_1(q,\rho)}{\partial q\,\partial} = \frac{\partial^2\pi_1(q,\rho)}{\partial\,\partial q} = \frac{rq}{(\alpha q + r\rho)}(\alpha P_s + h) \tag{18}$$

We must resolve the following optimization problem in order to increase net profit in relation to time spent.

Optimize π1 (q, ρ)

Assuming

$$\left\{\left(\frac{r(\alpha P_s + h)^2}{(r\rho + \alpha q)^2}\right)\left[2K_1 r^{\alpha_1}\rho + Kq^{\gamma-1}(\gamma-1)(\gamma-2) + \frac{\rho r q^2}{(r\rho + \alpha q)^2} - rq^2\right] + 2K_1 r^{\alpha_1} K(\gamma-1)(\gamma-2)q^{\gamma-3}\right\} > 0\ \forall q, \rho > 0 \tag{19}$$

Finding the values of q and ρ that, when optimized, produce the optimum results is the aim of optimizing the function of net profit. Determining the optimal values for ρ and q, which have an impact on the unit profit function, may be challenging. Various numerical methods can be employed to address this constrained optimization problem. In this specific case, however, To find the values that maximize the advantages in accordance with the selection criteria, we utilize the LINGO 13.0 program.

D. Numerical Example

Let's explore a scenario involving inventory with the following specific parameters

- The cost of placing an order, K, is 200 rupees.
- One unit costs $5 to store for one unit of time, h.
- 1200 units are needed for every unit of time, or r, as it is known.
- There is a fee of 100 rupees per unit, or c.
- The unit price, or Ps, for sale is Rs. 125.
- 0.5 rupees is the unit cost of shortages, denoted by γ.
- The discount rate, α, is 5%.
- The value of K1is 200, and α1 is 1.0.

LINGO 13.0 software was used to optimize equation (12) and find the best values for q∗ and ρ∗. In comparison to other models that were tested, the current model produces higher values for net profit, cycle time, order amount, units lost as a result of deterioration, as demonstrated by the analysis findings, which are presented in Table 1. This suggests that the present method, which accounts for variable ordering, deterioration-related unit losses, and promotional effort costs, helps managers make better decisions when faced with uncertainty.

Table I: Ideal Values for the Suggested Model

Model	Deterioration	Iteration	q**	tc*	L*	OC	ρ*	PE Price	π2 (q, ρ)	π (q, ρ)
Sharp	Indeed	115	25517.82	2.355663	1473.291	1.25201	8.50593	135213.6	171240.2	72692.1

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Model	Deterioration	Iteration	q**	tc*	L*	OC	ρ*	PE Price	π2 (q, ρ)	π (q, ρ)
Crisp	Yes	-	220	0.183	1.002213	200	-	-	5074.568	27806.1
%Change	-	-	99.1379	92.2315	99.9320	-15874.3	-	-	97.03657	61.7481
Crisp	No	41	309.839	0.258	-	200	-	-	7345.968	28450.8
%Change	-	-	98.7858	89.0477	-	-15874.3	-	-	95.71014	60.8612

E. Sensitivity Analysis

Exploring how the discount rate α affects retailer behavior presents an intriguing opportunity for further investigation. Based on the computational data presented in Table 2, several managerial insights have emerged:

- The amount of inventory lost as a result of degradation (shown by α) is correlated with the duration of the cycle of replenishing.
- Conversely, however, there is an inverse relationship between α and the ideal replenishment amount, total units lost each cycle, promotional effort, cost of promotional effort, Both the average and total profits per unit of time are shown. These measures show a drop at first, then a rise in later periods.
- At first, the cost of optimal variable ordering is higher, but it gradually decreases as α rises.

The link between the quantity of orders q and the units lost each cycle as a result of degradation (L) is depicted in Figure 2, while Figure 1 illustrates the relationship between the dynamic setup cost (OC) and the number of orders (q). These visual representations provide a clear depiction of these associations.

Table II: α Sensitivity Analysis

α%	Iteration	q**	tc*	L*	ρ*	OC	PE Price	π2 (q, ρ)*	π (q, ρ)*	%change in π2 (q, ρ)*
.04	107	31252.75	2.634015	1617.499	9.375818	1.131321	168370.4	208572.6	79184.30	-21.8012
.10	94	11881.95	1.541512	892.2890	5.940955	1.834789	58591.28	82305.11	53392.45	51.93587
.15	111	7024.196	1.145677	586.2808	4.682760	2.386336	32550.53	50224.19	43838.01	70.67033
.30	74	857.3087	0.6473005	80.54807	1.0000	6.830640	0.0000	10014.91	15471.81	94.15154
.50	123	1267.706	0.4096291	125.3938	2.323875	5.617211	4206.346	10552.52	25761.15	93.83759
.90	70	557.4794	0.2362594	57.17051	1.764688	8.470628	1403.395	5061.191	21422.18	97.04439

It is possible that by analyzing the fundamental factors K1, α1, h, r, c, Ps, γ, and one might gain useful insights about the behavior characteristics of retailers.

Let us assume that tc is the cycle length, q is the ideal amount for replenishment, ρ is L is the ideal number of units lost due to deterioration, and the optimal promotional effort factor. The cost of the promotional effort (PE) is not much impacted by the parameter K. Nonetheless, there is optimization of both the ideal The ideal average profit per unit per cycle (π) and net profit per unit per cycle (π1) are found. However, K has very little effect on the setup expenses for the variable OC.

As opposed to this, the parameter h significantly affects a number of variables, including the optimum net profit per unit per cycle (π1), the ideal average profit per unit per cycle (π), the length of the replenishment cycle (tc), the cost of promotional effort (PE), and the appropriate replenishment amount (q). Nevertheless, h has little effect on the OC variable's setup cost.

Lastly, neither the ideal promotional effort factor (ρ) nor the replenishment cycle time (tc) are much affected by the parameter r. R does, however, affect the units lost to deterioration (L) and the ideal replacement quantity (q). R has a significant impact on both the ideal The cost of the promotional effort (π1) and the average profit per unit each cycle (π). Furthermore, rrr has a minor impact on the OC variable's setup costs.

The optimum net profit per unit per cycle (π1), the variable setup cost (OC), and the optimal average profit per unit per cycle (π) and the cost of promotional effort (PE) are all impacted by the parameter c. It also influences the appropriate quantity for replenishment (q), the ideal promotional effort factor (ρ), the units lost to deterioration (L) and the duration of the replenishment cycle (tc).

K1 has no effect on the parameters γ and tc (how long the cycle of replenishing lasts). γ, however, has a significant impact on the ideal units lost to degradation (L), the ideal promotional effort factor (ρ) and replenishment quantity (q). γ has no effect on the expense of the promotional effort (PE), the optimal average profit per unit per cycle (π), or the ideal net profit per cycle (π1).

The length of the replenishment cycle (tc) remains unaffected by α1. In contrast, ρ, q, L, the variable setup cost (OC), the cost of the promotional effort (PE), K1 affects both the ideal The ideal net profit per unit per cycle (π1) and average profit per unit per cycle (π) are found.

Additionally, α1 affects the ideal net profit per cycle (π1), the variable setup cost (OC), the cost of the promotional effort (PE), and the ideal average profit per cycle (π).

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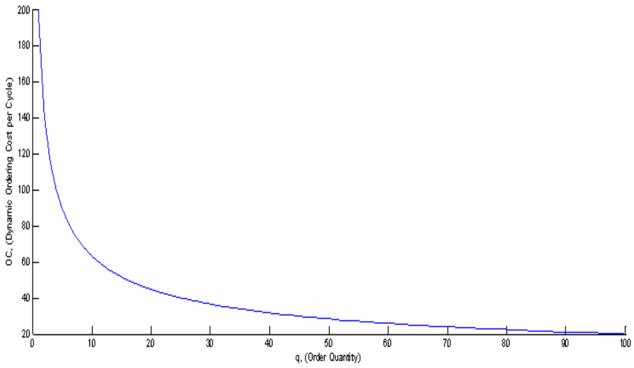

Figure 1. Plotting the Dynamic Ordering Cost (OC) and Order Quantity (q) in two dimensions.

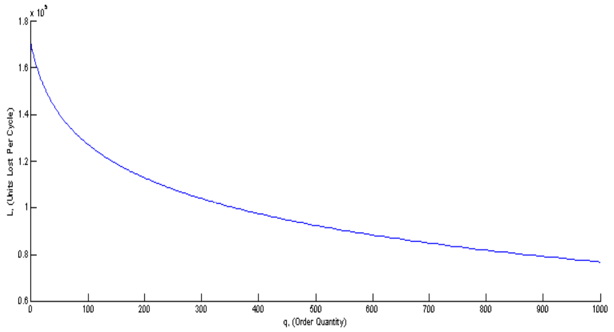

Figure 2. Units Lost per Cycle (L) and Order Quantity (q) in a Two-Dimensional Plot

F. Comparative Analysis

According to the data, the retailer always factors in the quantity of unused inventory when determining the portion of available inventory that is lost as a result of EOQ model deterioration. This approach aims to maximize potential net profit while minimizing the required time.

Figure 3 provides a comparison of order quantities and net profits among various related EOQ models, highlighting the results of this analysis. Based on the information in Table 3, Table 4 and Table 5, it can be concluded that, under all conditions, (q*) is consistently lower than q*, and π_{1*} is consistently lower than π1. This suggests that the current model performs better compared to the other EOQ models.

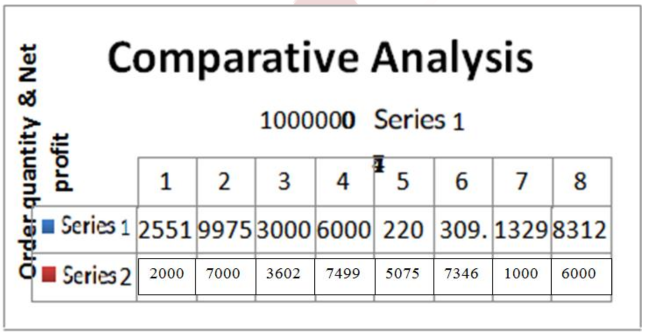

Figure 3. Comparative Evaluation of Net Profit and Order Quantity in Various EOQ Models

Table III: Q** and Q* comparative analysis (q** < q*)

OC	PE	q**	q*
Changeable	Right now	25517.82	99750.04
Changeable	-	3000.082	6000.058
-	Right now	13297.8	83125.0
-	-	220	309.839

Table IV: π1* and π1 (π1* < π1) Comparative Analysis

OC	PE	q**	q*
Changeable	Right now	171240.2	660936.9
Changeable	-	36024.74	74997.42
-	Right now	127738.9	550581.2
-	-	5074.57	7345.97

Table V: π1* and π1 (π1* < π1) evaluations in relation to **α%**

α%	π1*	π1
.04	208572.6	660936.9
.10	82305.11	660936.9
.15	50224.19	660936.9
.30	10014.91	660936.9
.50	10552.52	660936.9
.90	5061.191	660936.9

III. CONCLUSION

Recent research into price promotions and sales has provided valuable insights. The effectiveness of a promotional campaign is key to calculating the associated promotional activity costs. This study explores the impact of promotional effort costs on a modified amount of inventory lost to degradation while it is in stock and the importance of variable ordering costs are both highlighted by the Economic Order Quantity (EOQ) model. It also examines how inventory conditions affect stock management.

The study introduces a new mathematical model that incorporates variable setup costs and compares it numerically with the traditional EOQ model. The comparison reveals that the modified model yields a higher net profit and the economic order quantity (q_{**}). Furthermore, compared to the traditional approach, the adjusted average profit per unit every cycle is higher. This is due to the fact that disposing of deteriorated inventory affects the mean earnings per unit for every cycle.

The research highlights how integrating considerations of deteriorated inventory and promotional costs broadens the model's applicability. The theoretical findings are supported by a numerical example, with additional data derived from sensitivity analysis of key parameters: K1, α1, ό, α1, Ps, h, r, and c. This complete strategy accounts for variable ordering costs, promotions, and the portion of available inventory that is lost to deterioration. The study employs a robust model to analyze promotions, deterioration, and variable ordering costs in a specific scenario.

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